The UNNS Hamiltonian Protocol (UHP): Energy, Phase Space, and Canonical Recursion

UNNS Research Notes

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Abstract

Following the UNNS Lagrangian Protocol (ULP), we develop the *UNNS Hamiltonian Protocol* (UHP). This framework introduces recursive phase space, defines canonical variables, and derives Hamilton's equations for recursion dynamics. The UHP provides conservation laws, bridges to quantization, and a symplectic foundation for recursive systems.

Contents

1 Motivation

The Lagrangian Protocol supplies an action principle for recursion. To analyze stability, conservation, and quantization, we require a Hamiltonian formalism. The UHP introduces phase space and Hamilton's equations, extending UNNS into a full dynamical discipline.

2 Recursive Phase Space

Definition 2.1 (Phase Space)**.** *For a nest $\mathcal{N} = (a_0, a_1, a_2, \dots)$ with coefficients a_i, the phase space consists of:*

$$\mathcal{P} = \{(a_i, p_i) \mid i \geq 0\},$$

where $p_i = \frac{\partial L}{\partial \dot{a}_i}$ are the conjugate momenta derived from the recursive Lagrangian.

Remark 2.2. *Here $\dot{a}_i$ denotes the recursive "time" derivative, i.e., the evolution step along recursion depth.*

3 Hamiltonian Function

Definition 3.1 (Recursive Hamiltonian)**.** *The Hamiltonian is defined as*

$$H(\{a_i\}, \{p_i\}) = \sum_i p_i \dot{a}_i - L(\{a_i\}, \{\dot{a}_i\}),$$

where L is the recursive Lagrangian.

Remark 3.2. *H encodes recursive energy: balance of growth and stability.*

4 Canonical Equations of Recursion

Theorem 4.1 (Hamilton's Equations for UNNS)**.** *The recursion dynamics satisfy*

$$\dot{a}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial a_i}.$$

Proof. Follows from Legendre transformation of the Lagrangian action principle. □

4.1 Symplectic Structure

Definition 4.2. *The phase space carries a symplectic form*

$$\omega = \sum_i da_i \wedge dp_i,$$

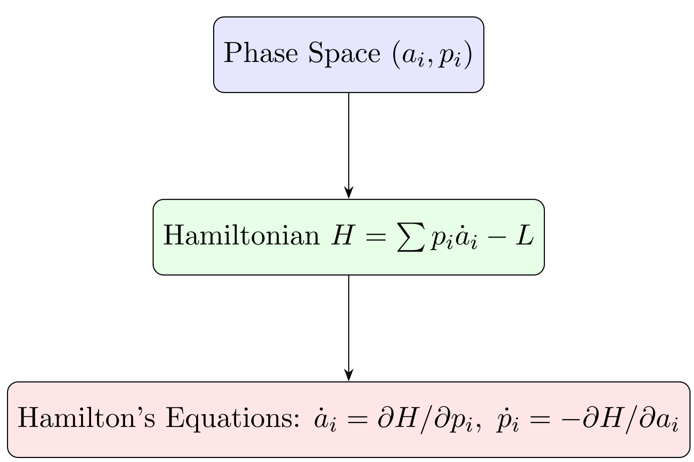
which is preserved under recursion flow.

Lemma 4.3 (Liouville's Theorem for UNNS)**.** *Recursive phase space volume is preserved:*

$$\mathcal{L}_{X_H} \omega = 0,$$

where X_H is the Hamiltonian vector field.

5 Diagrammatic Overview



6 Applications

6.1 Mathematics

- Establishes recursion symplectic geometry.
- Opens path to invariant theory for UNNS dynamics.

6.2 Physics

- Encodes recursive energy conservation laws.
- Sets the stage for UNNS quantization.

6.3 Computation

- Enables simulation via symplectic integrators.
- Provides diagnostics for recursive stability.

7 Conclusion

The UNNS Hamiltonian Protocol completes the variational structure of UNNS, providing phase space, energy conservation, and symplectic invariants. It paves the way for quantization of recursive systems, positioning UNNS as a substrate for both mathematics and physics.